

The Dimensionality of Congressional Voting Reconsidered

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Reviewer’s Appendix: Estimating the Dimensionality of Roll Call Data

Eigenvalues

While eigenvalues are computationally convenient quantities to examine when assessing the dimensionality of roll call data, they do not provide nor rely on measures of fit of OC or NOMINATE. The eigenvalues reported by OC and NOMINATE are the eigenvalues of the double-centered squared distance matrix. Specifically, one can replicate the same eigenvalues by

1. Dropping lopsided votes.¹ Suppose this leaves q votes by p legislators.
2. Calculate the $p \times p$ agreement score matrix, where the ij^{th} entry is the fraction of votes where legislators i and j voted the same way.
3. Calculate the distance matrix by taking $1 -$ (each entry).
4. Calculate the squared distance matrix by squaring each entry.
5. Double-center the matrix by subtracting the row and column means and adding the overall mean to each entry, then dividing by -2 .
6. Calculate the eigenvalues of this matrix. Note that there are p eigenvalues, the same as the number of *legislators*, not the number of votes.

Other than using a different default threshold, there is no difference between the eigenvalues reported by OC and NOMINATE. The eigenvalues have nothing directly to do with the models assumed by OC or NOMINATE; rather, they depend on the assumptions of principal component analysis.

Why use eigenvalues instead of some more direct measure of dimensionality? It’s fast, and it works,² at least when the number of votes is large. However, it is not clear that it works when the number of votes is smaller, such as when we focus on the votes for a single bill.

Harding (2008) shows that eigenvalue analysis of finite samples bias toward the identification of unidimensionality. This suggests that our eigenvalue-based analysis of dimensionality *underestimates* the number of dimensions. To be certain, we verified our results using an alternative method that is based on the reduction of classification errors as we assume a higher dimensional OC model.

¹NOMINATE defaults to dropping roll calls where the minority is less than 2.5% of the legislators casting votes; OC defaults to dropping votes where the minority is less than 0.5%. OC can use a smaller default threshold because the algorithm is less prone to crashing given nearly unanimous votes.

²See Poole (2005, p. 144) and Poole, Sowell, and Spear (1992).

Marginal Proportional Reduction in Error

Consider, for example, a Senate vote that is 60 ‘aye’ and 40 ‘nay’. A “zero” dimensional model that predicts all senators vote with the majority will have 40 classification errors, exactly the size of the minority. Suppose a 1-dimensional OC analysis correctly classifies 75 senators’ votes, and a 2-dimensional OC analysis correctly classifies 85 votes.

Aggregate proportional reduction in error (APRE) is defined as

$$\text{APRE} = \frac{\sum_{j=1}^q [\text{minority vote} - \text{classification errors}]_j}{\sum_{j=1}^q [\text{minority vote}]_j}$$

The reasons for using APRE instead of just measuring the fraction of votes classified correctly are twofold. First, we don’t want to give “credit” to the 1-d model for explaining the 60 votes of the majority, because those can be classified correctly just by noting that the measure passed, so we only count the improvement in the number of correctly classified votes, minority vote – classification errors. Second, the most improvement the 1-d model can show is to explain all of the 40 minority votes misclassified in the simpler model; dividing by the number of minority votes puts APRE on a $[0, 1]$ scale.

Let’s compare using the fraction classified correctly and APRE in our example:

$$\begin{aligned} \text{fraction classified correctly, 0-d model} &= .600 \\ \text{fraction classified correctly, 1-d model} &= .750 \\ \text{fraction newly classified correctly, 1-d model} &= .750 - .600 = .150 \\ \text{APRE}_1 &= \frac{40 - 25}{40} = \frac{3}{8} = .375 \end{aligned}$$

The APRE for the 1-d model is .375, which is the fraction of votes that were misclassified under the 0-d model but that are correctly classified under the 1-d model. Is this a good measure? That depends on the inference one wishes to draw based on the measure. APRE has face and construct validity when used to describe how much better the 1-d model is than the 0-d model because it chooses quantities for both the numerator and denominator relating to the inference in question (the difference between the two models.)

Now consider the 2-d model. Poole and Rosenthal (1997, 2007, chapter 3) measure the contribution of adding a second dimension by calculating the APRE for both models and subtracting.

$$\begin{aligned} \text{APRE}_1 &= \frac{40 - 25}{40} = \frac{3}{8} = .375 \\ \text{APRE}_2 &= \frac{40 - 15}{40} = \frac{5}{8} = .625 \\ \text{APRE}_2 - \text{APRE}_1 &= \frac{2}{8} = .250 \end{aligned}$$

The problem with $\text{APRE}_2 - \text{APRE}_1$ is that it is a measure of how much *more* the 2-d model improves on the 0-d model than the 1-d model does; it is not a measure of how much the 2-d model improves on the 1-d model. For the same reason, the 1-d model is not a .750 improvement on the 0-d model, that’s just the fraction correctly classified.

To measure the improvement of 2-d over 1-d, we want to use the quantity

$$\frac{25 - 15}{25} = \frac{2}{5} = .400$$

which is the fraction of the classifications the 2-d model could make correctly above and beyond those classified by the 1-d model. Generalizing APRE, we define the **marginal proportional reduction in error** from model A to model B as

$$\text{MPRE}_{AB} = \frac{\sum_{j=1}^q [\text{errors by A} - \text{errors by B}]_j}{\sum_{j=1}^q [\text{errors by A}]_j}$$

In terms of APREs, this can be calculated using

$$\text{MPRE}_{AB} = \frac{(1 - \text{APRE}_A) - (1 - \text{APRE}_B)}{1 - \text{APRE}_A}$$

References

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