Revealing Preferences:
Empirical Estimation of a Crisis Bargaining Game with Incomplete Information

by

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Question

What are the sources of state preferences in international crises?

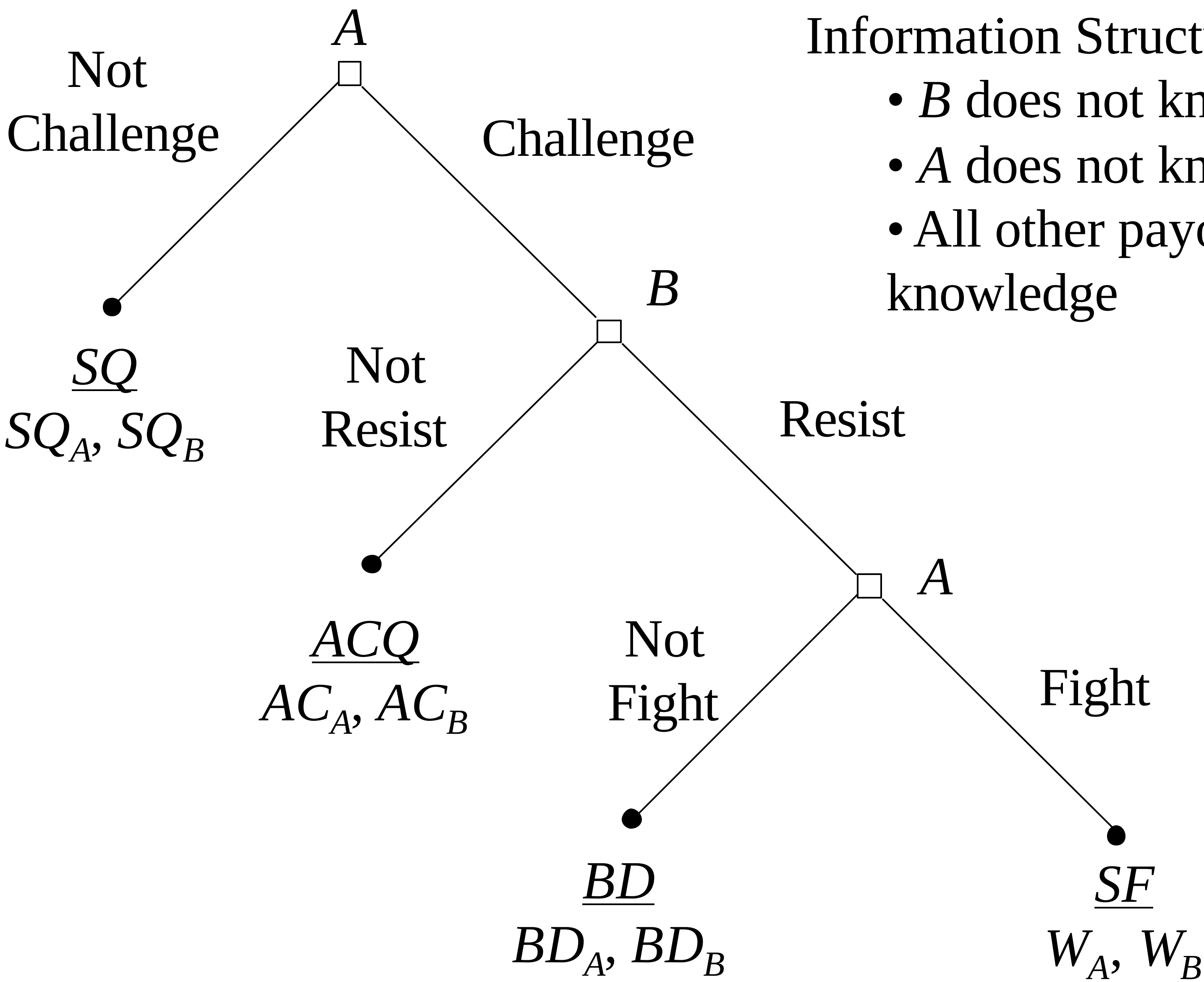
Possible hypotheses:

- Military superiority makes war more attractive
- Territorial contiguity increases opportunities for conflict and ability to bring military force to bear (e.g., Huth 1996)
- Commercial interdependence increases the value of peace (e.g., Russett and Oneal 2001)
- Democratic institutions
  - increase sensitivity to the costs of war (Kant 1795)
  - increase war-fighting ability Reiter and Stam 2000)
  - increase the "audience costs" associated with backing down from a threat (Fearon 1994)

Method

- Revealed Preferences: Use observational data on state choices during crises to infer…
  - the distribution of preferences over the possible outcomes
  - the effect of covariates (independent variables) on those preferences.
- Fully structural strategic model:
  - The empirical estimator is built directly from a crisis bargaining game with incomplete information.
  - Outcome probabilities are derived from the perfect Bayesian equilibrium of the game.

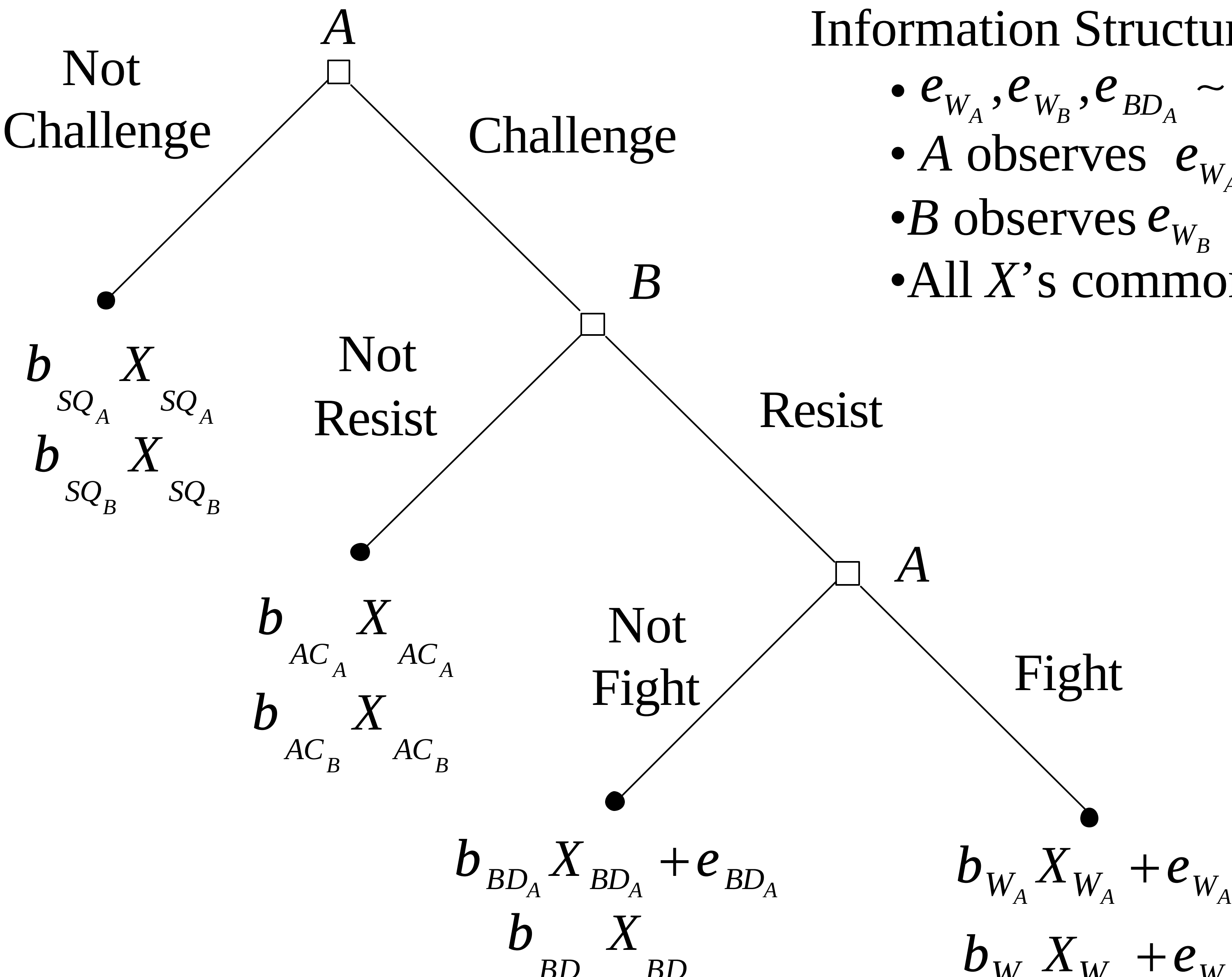
The Crisis Bargaining Game



Information Structure:

- *B* does not know W_A and BD_A
- *A* does not know W_B
- All other payoffs common knowledge

From Theoretical to Empirical Model



Information Structure:

- $e_{W_A}, e_{W_B}, e_{BD_A} \sim$ Normal $(0, s)$
- A observes e_{W_A}, e_{BD_A}
- B observes e_{W_B}
- All X's common knowledge

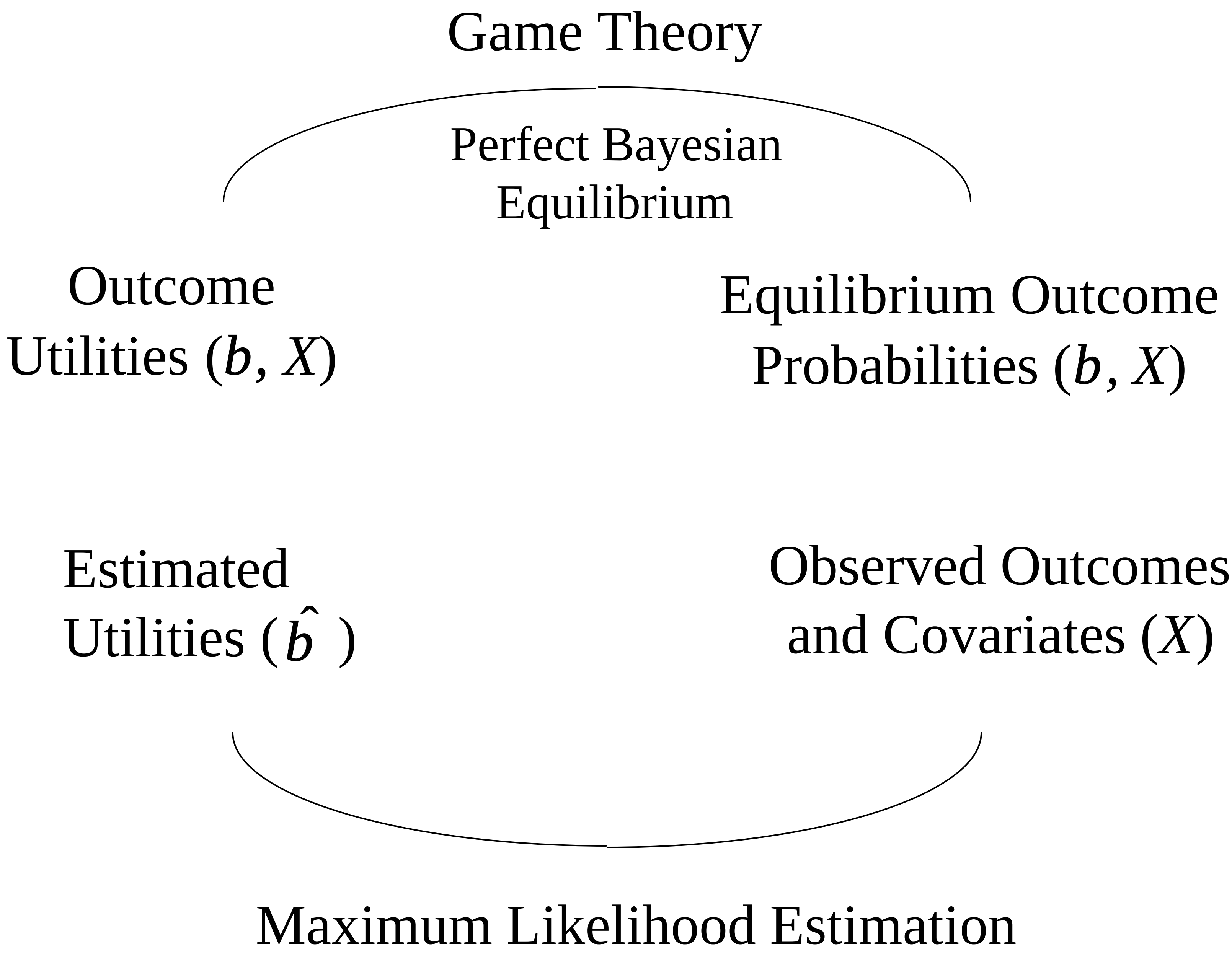
Perfect Bayesian Equilibrium

• All strategies are sequentially rational given the player's beliefs and the other player's equilibrium strategies.

• Beliefs are updated according to Bayes' Rule.

B updates about *A*'s type from the fact that *A* has made a challenge: the distribution of e_{W_A}, e_{BD_A} given a challenge is not the same as the prior distribution.

A knows that its decision influences *B*'s beliefs and takes this into account in determining its equilibrium strategy.


Game Theory
Perfect Bayesian Equilibrium
Outcome Utilities (b, X)
Equilibrium Outcome Probabilities (b, X)
Estimated Utilities ($\hat{b}$)
Observed Outcomes and Covariates (X)
Maximum Likelihood Estimation

Caveat

Not all interesting hypotheses about crisis behavior are about how covariates influence outcome utilities:

1. Sartori (2002): Past behavior influences credibility of current threats.

2. Schultz (1998): Democracy in challenger leads to a different game structure, with a strategic opposition party.

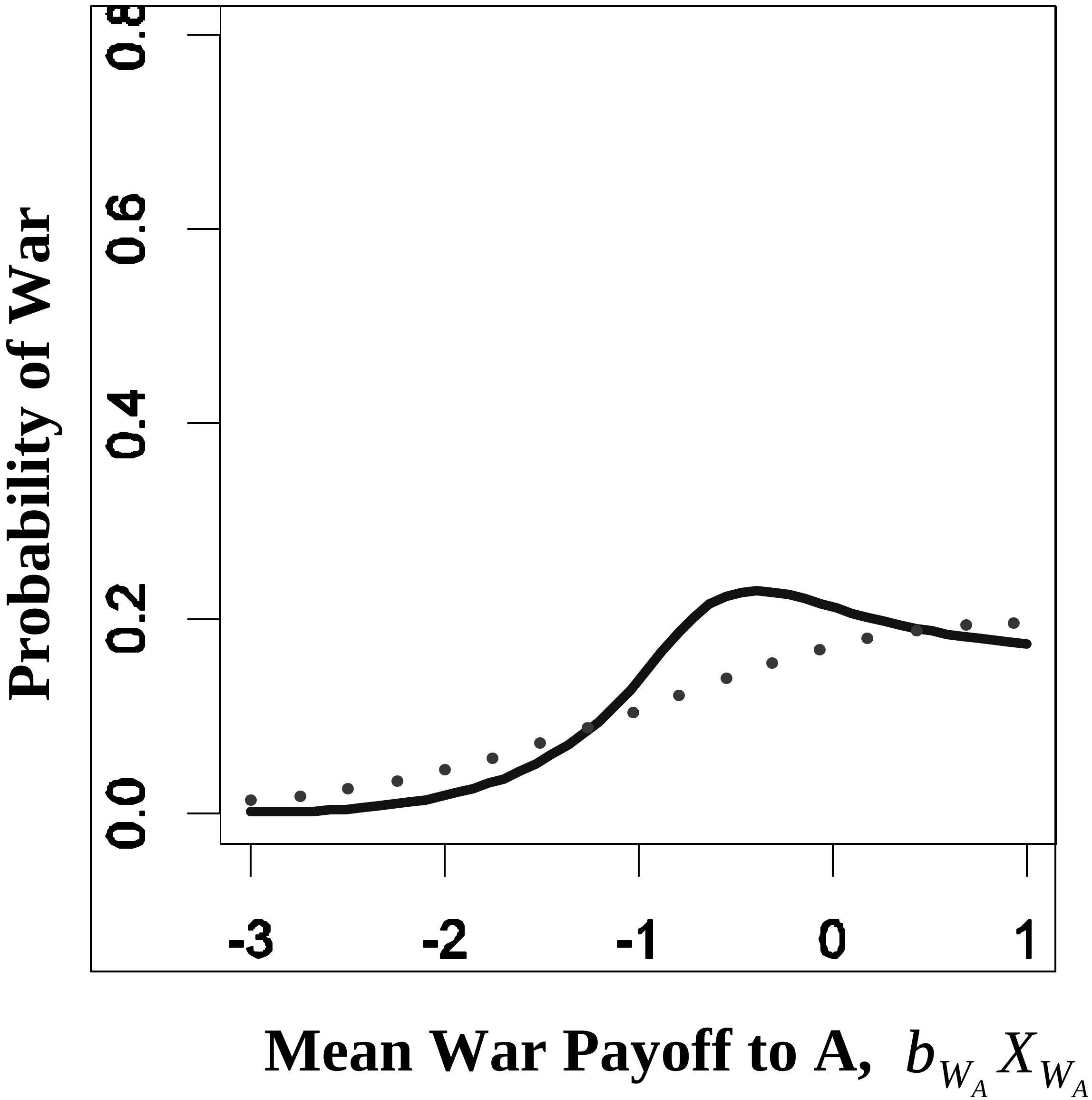
3. Transparency arguments: Perhaps democracy affects the variance of the private information?

Why Bother?

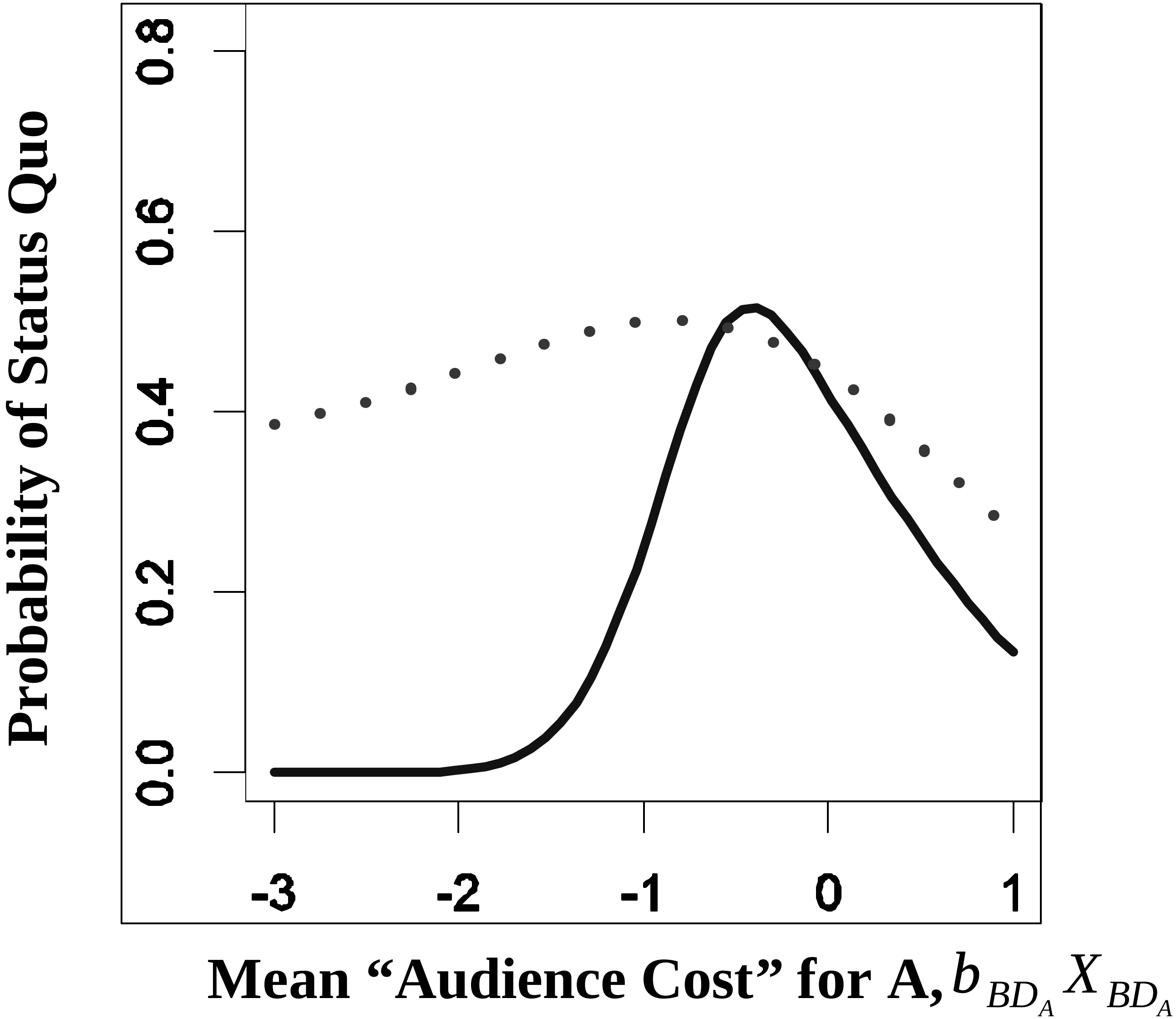
1. More accurately capture the mechanism that is believed to generate the data.

2. Strategic interaction can lead to non-monotonic relationships and other oddities that are not easy to capture in linear models (esp., Signorino 1999)

Non-Monotonic Effects



Non-Monotonic Effects (cont.)



Why Bother?

1. More accurately capture the mechanism that is believed to generate the data.

2. Strategic interaction can lead to non-monotonic relationships and other oddities that are not easy to capture in linear models (esp., Signorino 1999)

3. Estimating full model rather than isolated pieces gives a more complete picture and test of relevant theories.

The Pitfalls of Standard Practice

Schultz, "Do Democratic Institutions Constrain or Inform?" (1999 *International Organization*):

Theoretical Prediction:

- If democracy decreases W_A, then democracy in A should *increase* the probability that B resists.
- If democracy decreases BD_A, then democracy in A should *decrease* the probability that B resists.

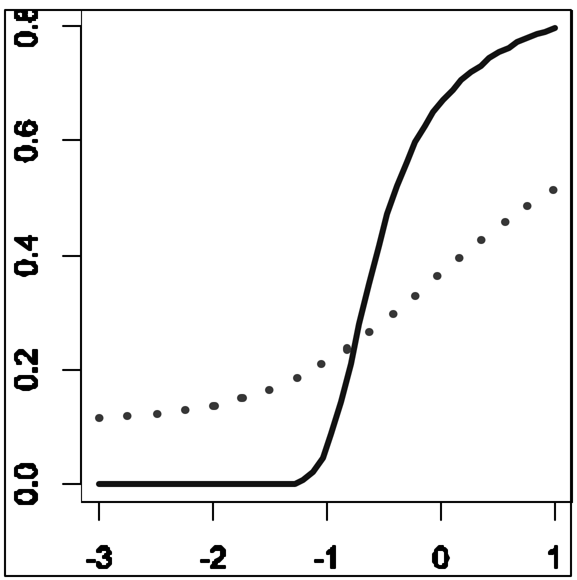
Empirical Result: Democratic initiators face lower probability of reciprocation in militarized disputes.

Conclusion: Democracy decreases BD_A.

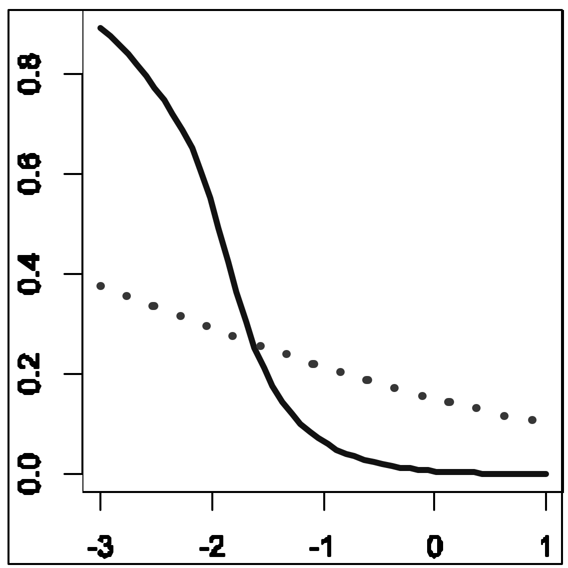
But…

A correlation between democracy in *A* and an increased probability of *ACQ* by *B* can be the product of more than one thing:

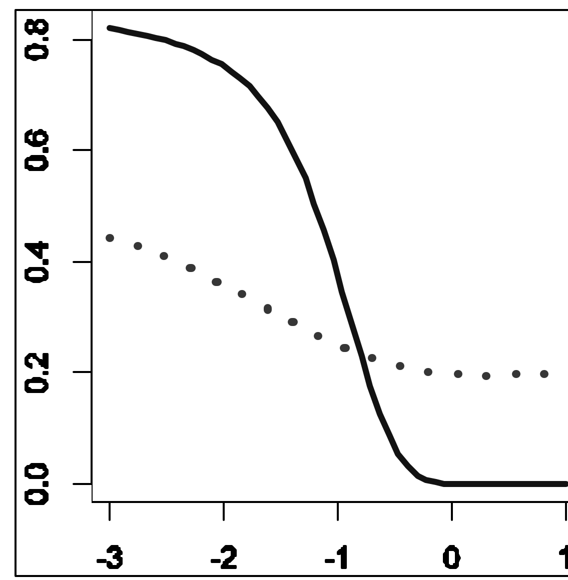
Equilibrium Probability of the ACQ Outcome



W_A



W_B



BD_A

The Identification Problem

Question: Given a set of observed outcomes and covariates, can you estimate a *unique* set of parameters $\hat{b}$?

Answer: In general, no, unless one imposes some *identifying restrictions*.

Identification in the Probit Model

We observe y_{it}, which equals 1 if war occurred in dyad i at time t, and zero otherwise.

Let y_{it}* measure the latent propensity for a war in dyad i at time t, and assume

$$y_{it} = \begin{cases} 0 & \text{if } y_{it}^* \leq c \\ 1 & \text{if } y_{it}^* > c \end{cases}$$

If $y_{it}^* = a + b\,X_{it} + e_{it}$ and $e_{it} \sim N(0, s)$, then

$$\Pr(y = 1) = \Pr(a + b\,X_{it} + e_{it} > c) = \Pr(e_{it} > c - a - b\,X_{it})$$

$$= \Phi\left(\frac{a - c}{s} + \frac{b\,X_{it}}{s}\right)$$

Identification in the Probit Model (cont.)

$$\Pr(y=1)=\Phi\left(\frac{a-c}{s}+\frac{bX_{it}}{s}\right)$$

Problem 1: Can add same constant to a and c and get same probability back.

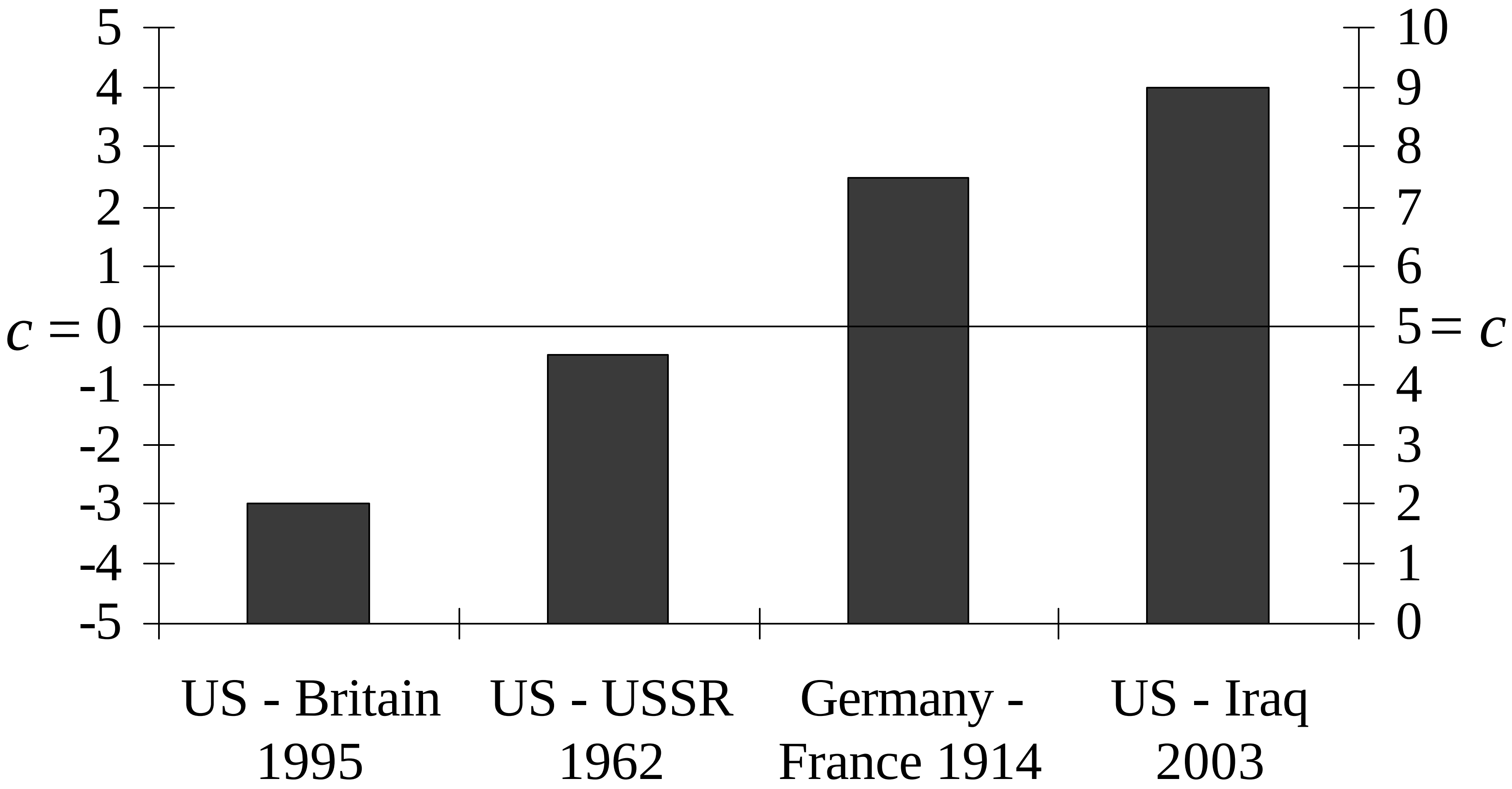
- If $\hat{a}$ and $\hat{c}$ are maximum likelihood estimates of a and c, then so are $\hat{a}+k$ and $\hat{c}+k$ for all k.
- Therefore, can't get unique estimates of both a and c (or a and c not uniquely identified).
- Usual solution: Assume c=0.

Problem 2: Can multiply b (and a) and s by same constant and get the same probability back.

- Therefore, b and s are not uniquely identified.
- Usual solution: Assume s=1.

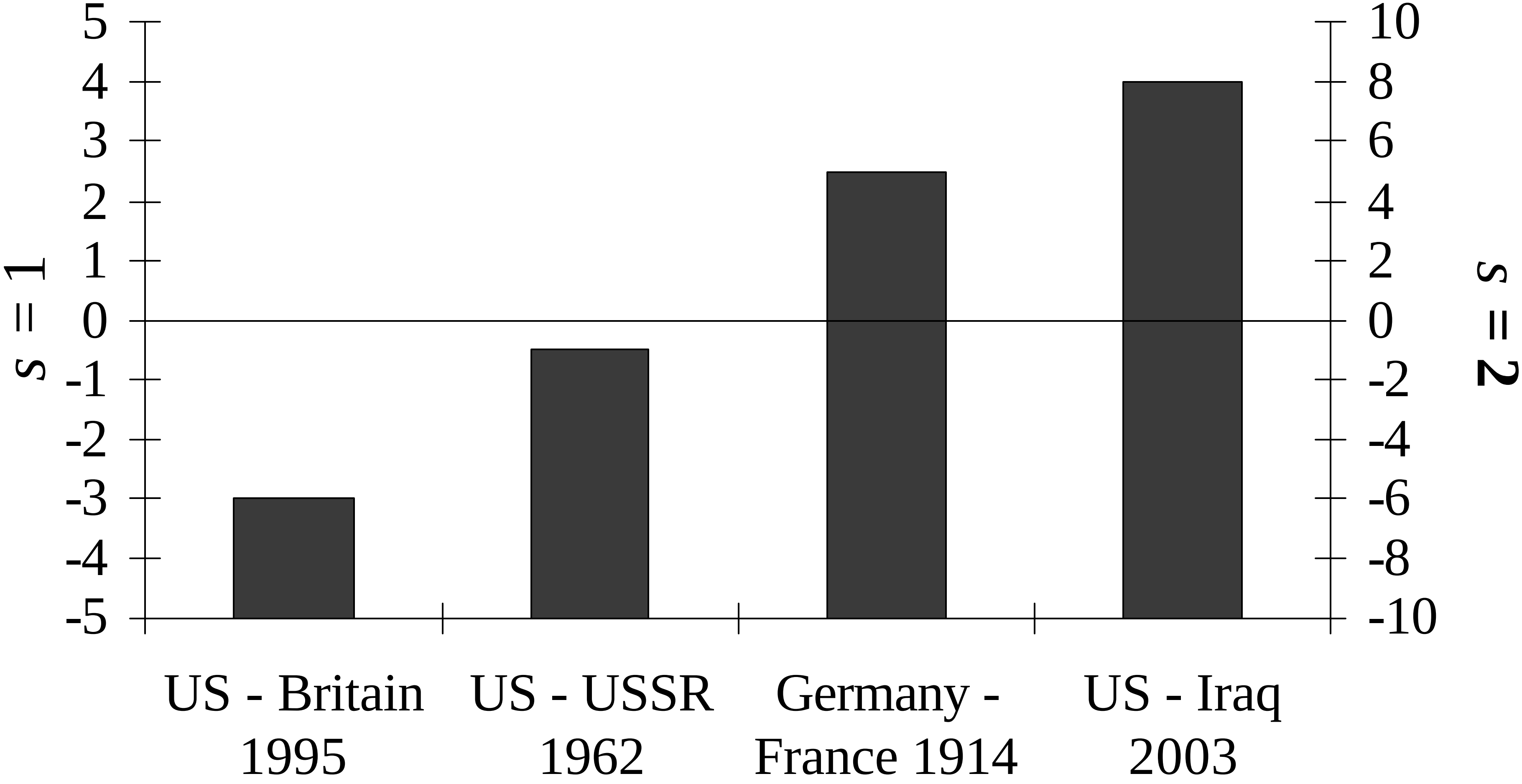
Why These Restrictions are Innocuous

Setting c determines the origin of the scale

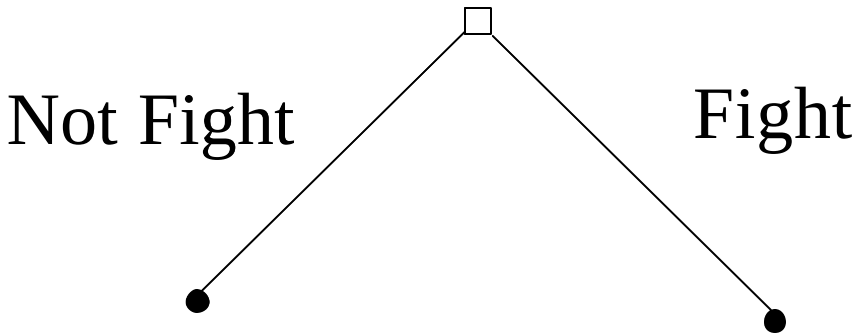


Why These Restrictions are Innocuous

Setting *s* determines the units of the scale



A Simple Random Utility Choice Model



$$U(\text{Peace}) = a_P + b_P X + e_P \qquad U(\text{War}) = a_W + b_W X + e_W$$

Assume $e_P, e_W \sim N(0,1)$

War if $U(\text{War}) > U(\text{Peace})$

$$a_W + b_W X + e_W > a_P + b_P X + e_P$$

$$e_P - e_W < a_W - a_P + (b_W - b_P)X$$

$$\text{Pr(War)} = \Phi\left(\frac{a_W - a_P + (b_W - b_P)X}{\sqrt{2}} \right)$$

Identification in the Random Utility Model

$$\Pr(\text{War}) = \Phi\left(\frac{a_W - a_P + (b_W - b_P)X}{\sqrt{2}}\right)$$

Problem 1: Constants a_W and a_P are not uniquely identified; only their difference is.

- Solution: Fix origin of utility scale by setting, e.g., $a_P = 0$.

Problem 2: Constants b_W and b_P are not uniquely identified; only their difference is.

- Solution:
  - Interpret results cautiously
  - Restrict b_P or b_W to 0

Why This Identification Problem is *Not* Innocuous

Suppose X indicates whether or the state is democratic, and you estimate that $b_W - b_P$ is negative. This is consistent with all of the following interpretations:

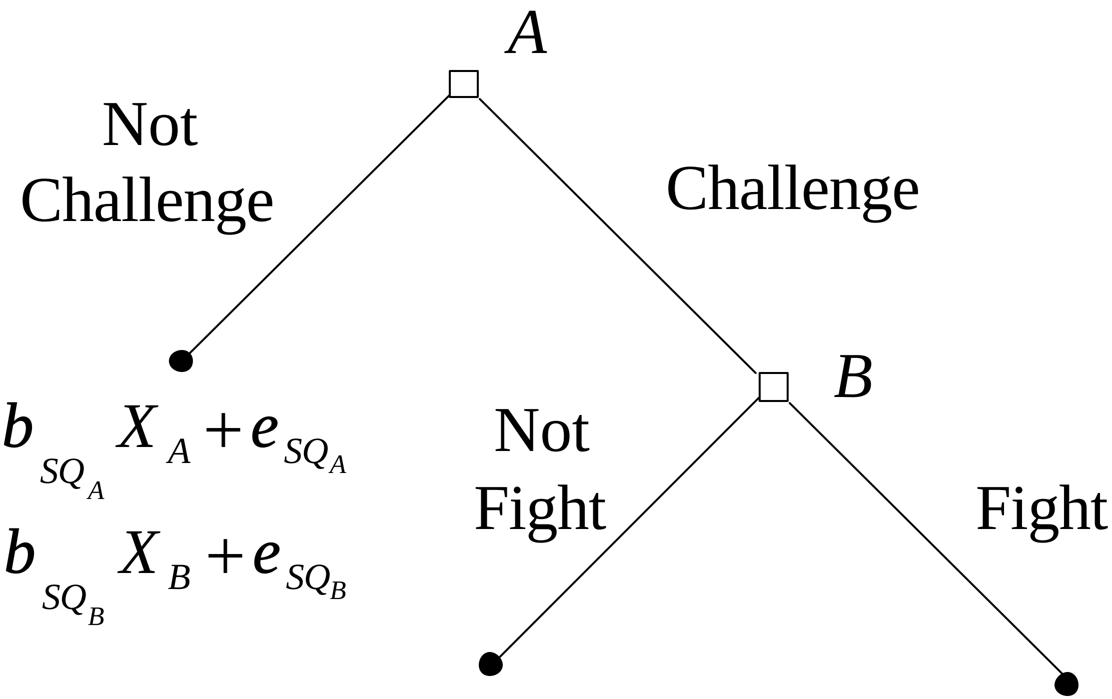
- Democracy increases *U*(Peace) and decreases *U*(War).
- Democracy increases *U*(Peace) and has no effect on *U*(War).
- Democracy increases *U*(Peace) and has a smaller positive effect on *U*(War).
- Democracy decreases *U*(War) and has no effect *U*(Peace).
- Democracy decreases *U*(War) and has a smaller negative effect on *U*(Peace).

To say anything about the direct effect of democracy on U(Peace) or U(War), you must be willing to make an identifying restriction:

- If you assume democracy has no effect on U(Peace), meaning $b_P = 0$, then you could conclude that democracy decreases U(War).
- If you assume democracy has no effect on U(War), meaning $b_W = 0$, then you could conclude that democracy increases U(Peace).

Whether or not these restrictions are justified is a *theoretical* matter, not an *empirical* matter! They cannot be tested empirically with observational data.

It Gets Worse…



$$SQ_A = b_{SQ_A} X_A + e_{SQ_A}$$

$$SQ_B = b_{SQ_B} X_B + e_{SQ_B}$$

$$AC_A = b_{AC_A} X_A + e_{AC_A}$$

$$AC_B = b_{AC_B} X_B + e_{AC_B}$$

$$W_A = b_{W_A} X_A + e_{W_A}$$

$$W_B = b_{W_B} X_B + e_{W_B}$$

Choice Probabilities:

$$\Pr(\text{B Fights})=\Pr(W_B > AC_B)$$

$$= \Phi\left(\frac{(b_{W_B} - b_{AC_B})X_B}{\sqrt{2}}\right) \equiv p_F$$

$$\Pr(\text{A Challenges})=\Pr\left[p_F W_A + (1-p_F)AC_A > SQ_A\right]$$

$$= \Phi\left(\frac{\left[b_{AC_A} - b_{SQ_A} + p_F(b_{W_A} - b_{AC_A})\right]X_A}{\sqrt{1+p_F^2+(1-p_F)^2}}\right) \equiv p_C$$

Outcome Probabilities:

$$\Pr(\text{SQ})=1-p_C$$

$$\Pr(\text{AC})=p_C(1-p_F)$$

$$\Pr(\text{War})=p_C p_F$$

Identification Problems

$$p_F = \Phi\left(\frac{(b_{W_B} - b_{AC_B})X_B}{\sqrt{2}}\right) \quad p_C = \Phi\left(\frac{\left[b_{AC_A} - b_{SQ_A} + p_F(b_{W_A} - b_{AC_A})\right]X_A}{\sqrt{1 + p_F^2 + (1 - p_F)^2}}\right)$$

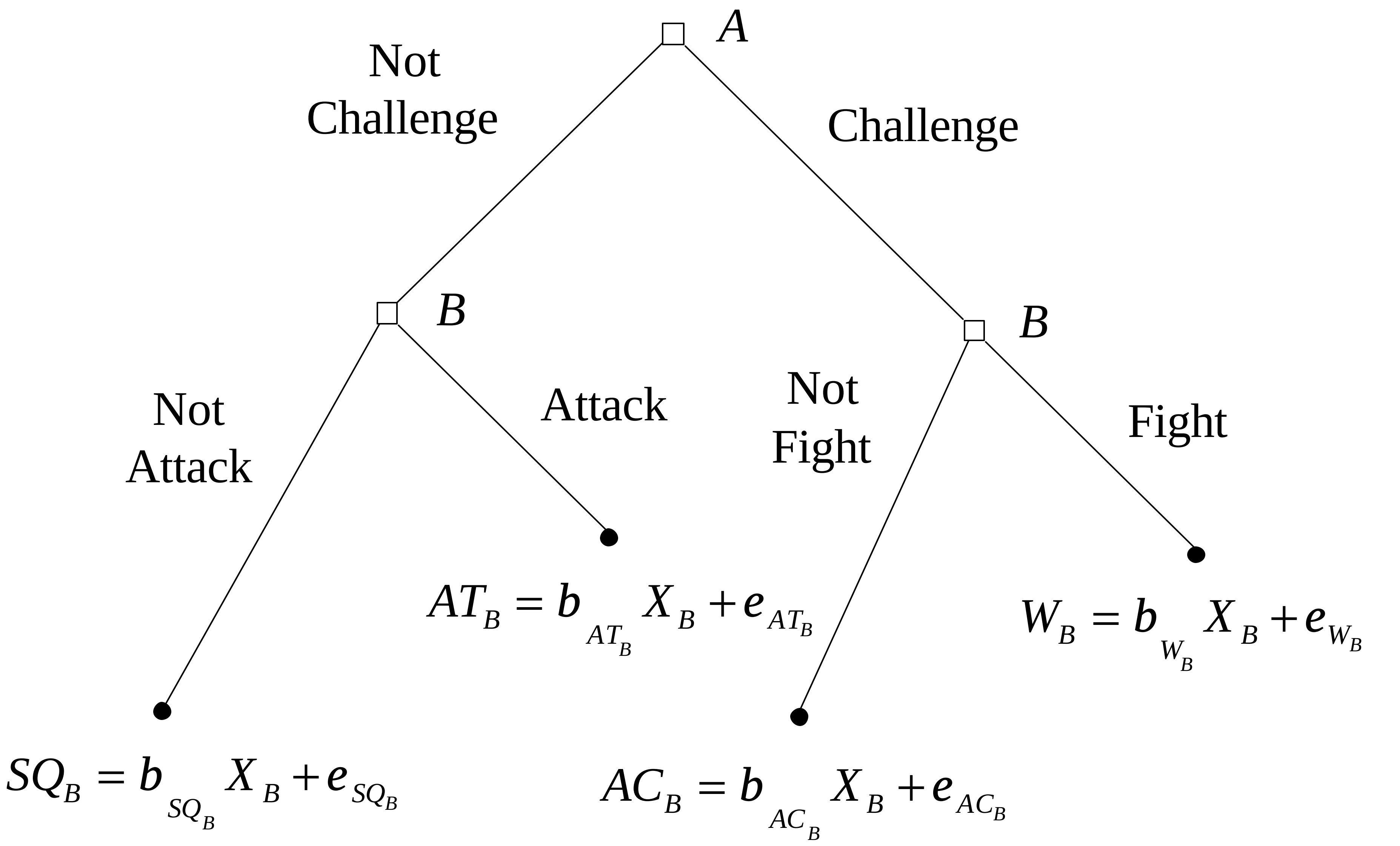
Problem 1: As before, *b* terms are not uniquely identified, because they appear as differences.

- To pin down direct effects, must set at least one *b for each player* to zero.
- For example, set $b_{SQ_A} = 0$, and

$$p_C = \Phi\left(\frac{\left[b_{AC_A} + p_F(b_{W_A} - b_{AC_A})\right]X_A}{\sqrt{1 + p_F^2 + (1 - p_F)^2}}\right)$$

Problem 2: None of the choice or outcome probabilities depend on b_{SQ_B}. Therefore, nothing can be learned about this parameter!

One More…


A
Not Challenge
Challenge
B
B
Not Attack
Attack
Not Fight
Fight
$AT_B = b_{AT_B} X_B + e_{AT_B}$
$W_B = b_{W_B} X_B + e_{W_B}$
$SQ_B = b_{SQ_B} X_B + e_{SQ_B}$
$AC_B = b_{AC_B} X_B + e_{AC_B}$

Identification Problems

$$\Pr(\text{B Fights}) = \Phi\left(\frac{(b_{W_B} - b_{AC_B})X_B}{\sqrt{2}}\right) \quad \Pr(\text{B Attacks}) = \Phi\left(\frac{(b_{AT_B} - b_{SQ_B})X_B}{\sqrt{2}}\right)$$

Problem 1: Cannot uniquely identify

- $\hat{b}_{W_B}$ and $\hat{b}_{AC_B}$, or
- $\hat{b}_{AT_B}$ and $\hat{b}_{SQ_B}$.

Problem 2: $\hat{b}_{W_B}$ and $\hat{b}_{AC_B}$ can move up and down together independently of $\hat{b}_{AT_B}$ and $\hat{b}_{SQ_B}$, and vice versa.

If $\hat{b}_{W_B}$, $\hat{b}_{AC_B}$, $\hat{b}_{AT_B}$, and $\hat{b}_{SQ_B}$ are ML estimates, then so are $\hat{b}_{W_B} + k$, $\hat{b}_{AC_B} + k$, $\hat{b}_{AT_B} + k'$, and $\hat{b}_{SQ_B} + k'$ for all k, k'

Why this is Problematic

Say X_B measures whether or not B is democratic.

Hypothesis: Democracy decreases the utility of an unprovoked attack more than it decreases the utility of fighting in response to a challenge, or $b_{AT_B} < b_{W_B} < 0$.

Unfortunately, you can only determine

- how democracy affects AT_B relative to SQ_B
- how democracy effects W_B relative to AC_B.

You cannot infer from this anything about relative magnitude of b_{AT_B} and b_{W_B} .

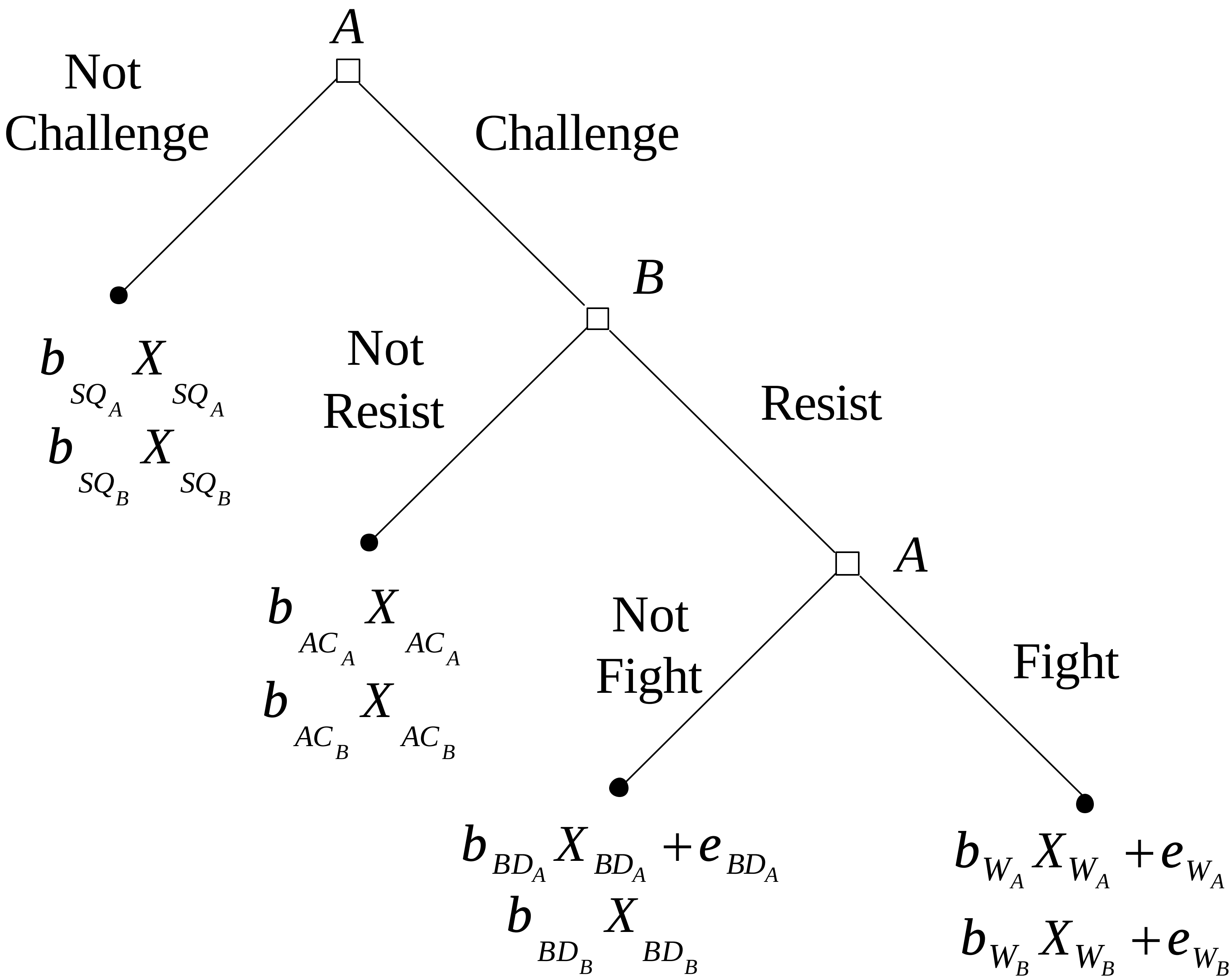
The Limitations of Revealed Preferences

1. Von Neumann-Morgenstern utilities are "cardinal utilities that are ordinal."
   - All choices are driven by differences in utilities.
   - Without restrictions, we can only estimate the effect of a covariate on the difference in utilities.

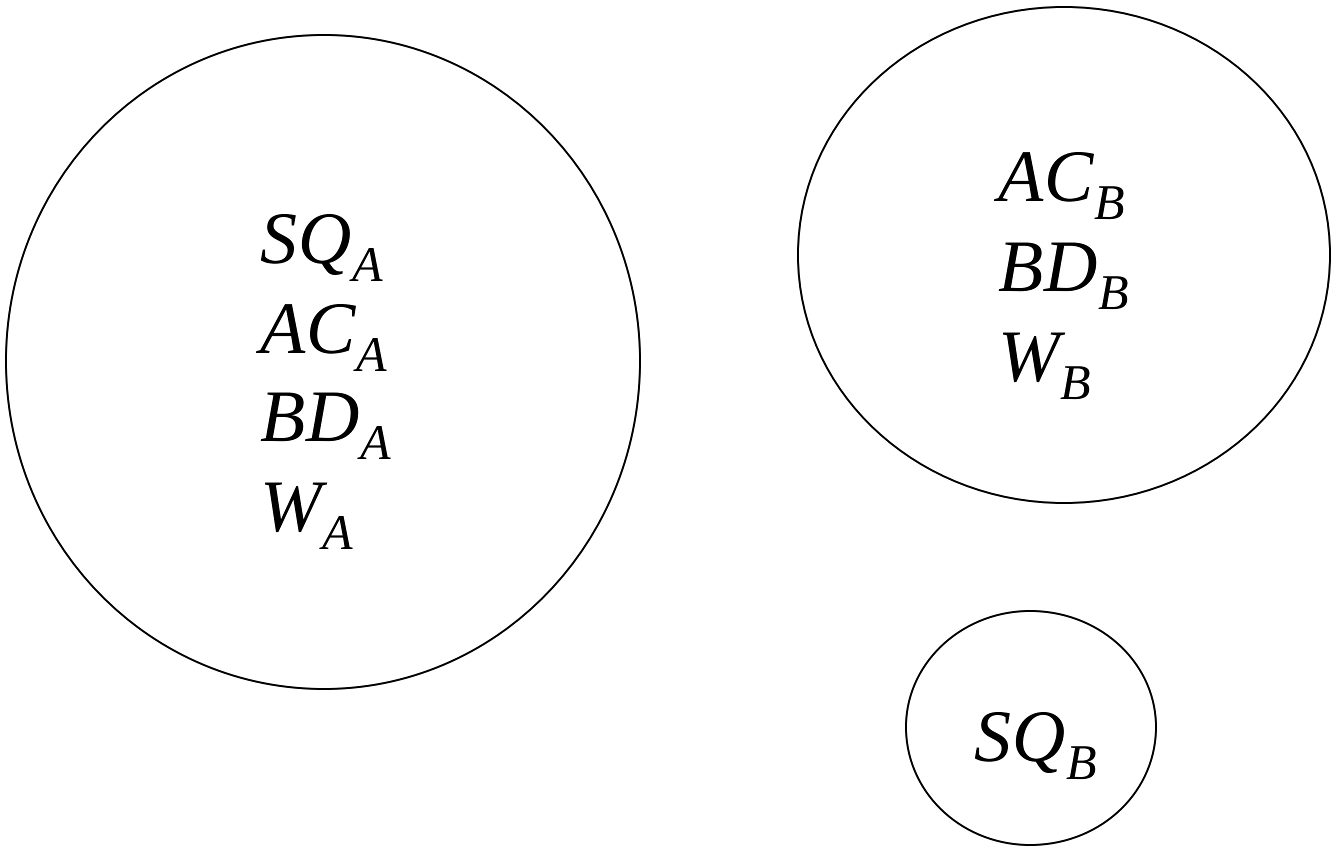
2. Relative utilities can be estimated only if the preference between them is revealed.
   - Utilities associated with outcomes that a player never gets to choose between cannot be measured on the same scale.

3. No interpersonal comparisons can be made either.

Back to Our Model



Three Subsets of Comparable Utilities



Identifying Restrictions Needed

Within each set of subset of comparable utilities:

- To estimate direct effects of a covariate, *X*, its effect on at least one utility within the subset must be set to zero.
- For example, the regime type of *A* can appear in the equations for W_A, BD_A, and AC_A, but not SQ_A at the same time.
- The constant has to be omitted from at least equation as well.

Across subsets of comparable utilities:

- No comparisons are possible without additional assumptions; e.g., $SQ_B = BD_B$.

Why We Need Theory

1. Theory informs the choice of theoretical model (i.e., extensive form of the game and the information structure).

2. Theories needed to motivate/justify identifying restrictions (i.e., what covariates belong in each equation, whose effects can be set to zero).

3. Theory directs data collection.

Data Needs

A data set whose observations are discrete plays of the crisis bargaining game, with a coding of

- which states were *A* and *B*,
- which outcome node of the game was reached,
- relevant covariates (e.g., relative power, regime type, interdependence, etc.).

Stay Tuned!