

**Strategic Voting under Plurality Rule:
When Does It Matter and Who should Vote Strategically?**

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ABSTRACT

This paper establishes the conditions under which strategic voting can change the election outcome, and identifies the voters who should vote strategically under simple plurality rule and the runoff rule. Three interesting findings can be drawn from the results. First, under simple plurality rule, strategic voting can change the election outcome only if the plurality winner is not the Condorcet winner or the Condorcet winner does not exist; under the runoff rule, strategic voting can change the election outcome only if the Condorcet winner is the plurality loser or the Condorcet winner does not exist. Second, under simple plurality rule, the Condorcet winner is not always the strategic voting outcome even if we assume that voters have complete information about the preference profile. Even worse, the Condorcet loser could win. Under the runoff rule, however, strategic voting ensures that the Condorcet winner will win if one exists. Third, strategic voting is less likely to occur under the runoff rule than under simple plurality rule.

Strategic Voting under Plurality Rule: When Does It Matter and Who should Vote Strategically?

Strategic voting under plurality rule refers to a voter deserting a more preferred candidate with a poor chance of winning for a less preferred candidate with a better chance of winning (Downs 1957, Farquharson 1969). According to Duverger, the behavior of strategic voting is the “psychological” effect of simple plurality system, which favors the two party system.¹ The formal-theoretic work on strategic voting under plurality rule focuses on the conditions under which Duverger’s law holds. Cox (1987) formalizes Duverger’s law by showing that “in almost all equilibria some voters vote strategically and that the marginal impact of strategic voting was to decrease the effective number of parties.” Palfrey (1989), elaborating on Cox’s model, shows that equilibria can be classified into two classes: *Duvergerian* equilibria (in which only two candidates receive votes and the third parties are devastated by strategic voting) and *non-Duvergerian* equilibria (more than two candidates receive votes). The intuition behind the non-Duvergerian equilibria is roughly as follows. Suppose one candidate is clearly ahead of the rest, but two candidates are nearly tied for second place. In this case, voters who supported the two trailing candidates cannot decide which one to desert, leaving more than two significant candidates in the field.²

¹ According to Duverger, the single-member simple plurality system also has a “mechanical” effect. Third parties or independent candidates may win many votes nationwide, but gain a plurality of the vote in very few electoral units.

² Myerson and Weber (1993) and Cox (1994) offer more general models of strategic voting under simple plurality rule. For a general survey of theoretical and empirical findings of strategic voting under plurality rule, see Cox (1997).

This literature on strategic voting under plurality rule ignores two important questions. First, under what conditions will strategic voting change the election outcome? Second, who should vote strategically? For example, are the supporters of the candidate with the fewest first-place votes most likely to vote strategically? Will the supporters of the first or the second highest vote getter ever find it in their interest to vote strategically? Niemi and Frank (1981) offered the first answer to these questions. Their definition of strategic voting, however, does not consider fully the possibility of strategic actions by voters. In this paper, I study the conditions under which strategic voting can change the election outcome and to identify the voters who should vote strategically.

1. Notations and Assumptions

Suppose there are three candidates, a , b , and c , competing for a single seat under plurality rule. I assume that voters have strong preference ordering over these candidates, that voters with the same preference ranking vote as a bloc, and that voters have complete information about which preference profile represents the true state of nature.³ Furthermore, I assume that all voters are short-term instrumentally rational (Cox 1987, Palfrey 1989). That is, voters care only about the outcome of the current election when they decide for whom to vote.

A preference profile is a distribution of voters over six possible preference rankings, $a > b > c$, $a > c > b$, $b > a > c$, $b > c > a$, $c > a > b$, $c > b > a$. So voters can be classified into six mutually exclusive voting blocs, n_1 to n_6 , according to their preference rankings over the

³ See Niemi and Frank (1981) for a justification of these assumptions.

three candidates. The number of voters in bloc n_i is denoted as $n(i)$, the total number of voters in blocs n_i and n_j are denoted as $n(i, j)$, and so on, $i, j = 1, 2, \dots, 6$.

Preference Profile

n1	n2	n3	n4	n5	n6
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

Given a preference profile, we can rank candidates in terms of the first-place votes. To make the problem of strategic voting under plurality rule nontrivial, I will assume no candidate is ranked first by a majority of the voters. Furthermore, for simplicity, I assume that no tie exists between any two candidates. We denote the first, second, and the third highest vote getters as a, b, c , respectively. Candidate a is the plurality winner and candidate c is the plurality loser. Given a preference profile of the voters, we can also determine the social preference relations among a, b , and c . If a Condorcet winner (the candidate who defeats the other two candidates in pairwise comparisons) exists, assuming strict social preference ordering, then one of the other two candidates is a Condorcet loser (the candidate who loses to the other two candidates in pairwise comparisons) and one the Condorcet neutral (the candidate who loses to the Condorcet winner but defeats the Condorcet loser). We can classify the social preference ordering reflected by the preference profiles into eight subsets, where $>$ indicates a pairwise comparison under majority rule.

1a: $a > b > c$ (The plurality winner is the Condorcet winner.)

1b: $a > c > b$ (The plurality winner is the Condorcet winner.)

2a: $b > a > c$ (The plurality winner is the Condorcet neutral.)

2b: $c > a > b$ (The plurality winner is the Condorcet neutral.)

3a: $b > c > a$ (The plurality winner is the Condorcet loser.)

3b: $c > b > a$ (The plurality winner is the Condorcet loser.)

4a: $a > b > c > a$ (The Condorcet winner does not exist.)

4b: $a > c > b > a$ (The Condorcet winner does not exist.)

2. Definitions of Strategic Voting

Farquharson (1969) defines strategic voting in terms of the elimination of dominated strategies. Under plurality rule, voting for one's least preferred alternative is dominated by the strategy of voting for one's most preferred alternative, so no voter will ever vote for his least preferred alternative. So in three-candidate elections under plurality rule, therefore, each voter has two strategies only: voting for either her first or second preferred candidate (Niemi and Frank, 1981, p.154). When Niemi and Frank (1981) apply Farquharson's definition of strategic voting in the context of three-candidate elections under simple plurality rule, they assume that the six voting blocs make their voting decisions simultaneously. They provide an example to show that even when the plurality winner is the majority winner, Farquharson's definition of strategic voting still leads to indeterminate outcomes (pp.156-7).

In light of the weaknesses of Farquharson's definition, Niemi and Frank (1981) develop a new definition of strategic voting. In their definition, voters make decisions sequentially. They define strategic voting in terms of three steps (pp.157-58):

1. Voters consider the current situation, a situation being a description of how all blocs vote and the outcome implied by that voting. The initial situation is sincere voting by all blocs.
2. Each bloc determines whether it can improve the outcome by altering its own vote while assuming that all other votes remain the same. It turns out that no more than two blocs can improve the outcome in this way.
- 3a. If no bloc can improve the outcome, the current situation is a Nash equilibrium, and the current situation contains the sophisticated strategies and the sophisticated outcome.
- 3b. If exactly one bloc can improve the outcome, it changes its vote accordingly, and the process reverts to step 1.
- 3c. If two blocs can improve the situation, a two-person game ensues, as described below. The result of the game is either that only one bloc will change its vote or the situation is what Luce and Raiffa (1957, pp. 90-91) call a "battle-of-the-sexes." In the former case, the one bloc changes its vote and the process reverts to step 1. In the latter case, the sophisticated outcome is indeterminate.

In the two-person game, each bloc has two strategies - voting its first or second choice - and it is assumed that all other players will not change their votes. Each bloc checks for dominated strategies in the manner described

earlier, and only strategies that are ultimately admissible in this game are used. Since it is a 2 X 2 game, there are no more than two simple reductions involved.

The definition of strategic voting by Niemi and Frank tries to capture how people act in real situations. Given a preference profile, if no one votes strategically, the plurality winner wins. So blocs n_1 and n_2 have no incentives to change their votes since the plurality winner is already their most preferred candidate. Blocs n_3 and n_5 have no incentives to vote strategically either. The plurality winner is their second most preferred outcome. By voting strategically for their second most preferred candidate, it would only add to the margin by which the plurality winner won. The two blocs, n_4 and n_6 , getting the worst outcome under the status quo are likely to be the first movers. Bloc n_4 would have incentives to vote strategically for c if $n(4,5,6) > n(1,2)$ and bloc n_6 would have incentives to vote strategically for b if $n(3,4,6) > n(1,2)$. If both blocs can change the outcome by voting strategically, then the two blocs will be playing a simultaneous game. After they move, other blocs can react to their actions, until no bloc has incentives to react.

Niemi and Frank's definition of strategic voting is actually a specification of the rules of the voting game. Blocs n_4 and n_6 move first, then other blocs decide whether to counter their moves. This move and counter-move sequence continues until no bloc has incentives to move anymore. Their definition of strategic voting, however, has two problems. First, voters in their game are not sophisticated. At step 3c, when n_4 and n_6 are playing the two-player game, they do not anticipate the reaction from n_3 or n_5 .

Consider the following example (this game is taken from Niemi and Frank, 1981, p.163, Game 7(b)). The plurality winner can be defeated if either n4 or n6 votes strategically. In the two-player game, n6's dominant strategy is to vote for b. According to their definition, the process reverts to Step 1. Bloc n5 then can counter n6's move by voting for a, which is n6's worst outcome. At this point, no block can further improve its outcome by voting strategically. The game ends and a wins. However, if n4 and n6 were sophisticated, n6 would not have voted for b. Instead, n4 and n6 would have voted for c since n3 could not alter this outcome by voting for a. The final outcome would be c, an outcome that blocs n4 and n6 both prefer over a.

Example 1

Preference Profile

n1	n2	n3	n4	n5	n6
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a
8	0	2	5	4	2

		n6	
		c	b
n4	b	a	b
	c	c	c

The second problem of Niemi and Frank's definition of strategic voting is that it assumes that n4 and n6 move first and the other blocs then try to counter their moves. Their definition does not allow other blocs to preempt the move by n4 and n6. Given the preference profile in Example 2, the plurality winner can be defeated if either n4 or n6 votes strategically. In the two-player game, n4's dominant strategy is to vote for c. According to their definition, the process then reverts to Step 1. At this point, no block can further improve its outcome by voting strategically. The game ends and c wins. However, if n3 can credibly commit to vote for a before n4 and n6 make their moves, by doing so it forces n4 and n6 into playing the battle-of-the-sexes game. The outcome of the game becomes indeterminate, which is better than c from n3's point of view. If n3's commitment is not credible, n3's dominant strategy is still to vote for b when n4 and n6 are playing the battle-of-the-sexes game after n3 commits to vote for a. Because if n4 votes for c and n6 votes for b, by voting for b, n3 can secure b since $n(3,6) > n(1,2)$.

Example 2

Preference Profile

n1	n2	n3	n4	n5	n6
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a
11	0	3	7	0	9

n6

		c	b
n4	b	a	b
	c	c	b

To correct the problem of anticipation in Niemi and Frank's definition, I use the subgame perfect equilibrium solution concept. That is, I assume that when voters decide for whom to vote, they will consider the consequence of their decision and reason backward. For example, if n4 decides to support c, obviously, n5 and n6 won't react to n4's move because c is their most preferred outcome. It is n3 who has incentives to counter n4's move because c is n3's least preferred outcome if uncountered. Therefore, n4 has to consider whether n3 has a viable counter. That is, if $n(1,2,3) > n(4,5,6)$, n4 cannot change the outcome by voting strategically even if $n(4,5,6) > n(1,2)$. So $n(1,2,3) < n(4,5,6)$ is the necessary condition for n4 to be a potential strategic voter. Also, $n(3,4,6) > n(1,2,5)$ is a necessary condition for n6 to be a potential strategic voter.

To correct the second problem of their definition, I study the effect of credible commitments on strategic voting by allowing n3 and n5 to move before n4 and n6 make their moves. I then compare the strategic voting outcomes when voters can and cannot make credible commitments.

In the next section, I analyze the strategic voting equilibrium under simple plurality rule for each of the eight possible preference profiles.

3. Strategic Voting Equilibrium under simple Plurality Rule

Cases 1a & 1b: The Plurality Winner is the Condorcet Winner

In the cases of 1a and 1b, the plurality winner is also the Condorcet winner. Strategic voting by n_4 or n_6 cannot change the outcome since by the definition of the Condorcet winner, candidate a defeats both b and c in pairwise comparisons, $n(1,2,5) > n(3,4,6)$ and $n(1,2,3) > n(4,5,6)$. So if n_6 votes strategically for candidate b , n_5 can counter n_6 's move by voting for a , or if n_4 votes strategically for c , n_3 can counter that by voting for a . In other words, strategic voting is inconsequential in cases 1a and 1b.

Remark 1a: In three-candidate elections, under simple plurality rule, if the plurality winner is also the Condorcet winner, then strategic voting is inconsequential and the plurality winner is victorious.

If we assume that voters have single peaked preferences on a uni-dimensional space, then the Condorcet winner must be positioned in between the other two candidates.⁴ If the plurality winner is also the Condorcet winner, no voters have preferences $b > c > a$ and $c > b > a$. In other words, no voters have incentives to vote strategically because blocs n_4 and n_6 are empty.

Remark 1b In three-candidate elections under simple plurality rule, if the plurality winner is also the Condorcet winner and if we assume single-peaked preferences, then no voters have incentives to vote strategically.

⁴ This is proved in Lemma 1. Lemma 1: If no candidate receives more than fifty percent of the votes, the Condorcet winner is positioned in between the other two candidates on a uni-dimensional space. Proof: Suppose the contrary is true. By definition, the Condorcet winner is preferred to the other two candidates by a majority of the voters in pairwise comparisons. This implies that the Condorcet winner must be

A rich set of empirical evidence is available to support Remark 1a. Abramson et al. (1995) show that in the 1968, 1980, 1992 American presidential elections, the plurality winners, Richard Nixon, Ronald Reagan, and Bill Clinton, were all Condorcet winners, and the third party or independent candidates, George Wallace, John Anderson, and Ross Perot, were all Condorcet losers. According to Remark 1a, supporters of the third party or independent candidates might have voted strategically, but the plurality winners could not be toppled because they were also the Condorcet winners. Therefore, supporters of the third party candidates had little incentive to vote strategically and could cast symbolic votes for their most preferred candidates.

The 1994 Taipei city mayoral election provides another example. In that election, three candidates were competing, Chen, Huang, and Jaw. Based on the election survey conducted by the Election Studies Center of the National Chengchi University, 61.2% of the respondents preferred Chen to Huang in the pairwise comparison, 55.1% preferred Chen to Jaw, and 53.7% preferred Huang to Jaw. This implies that Chen was the Condorcet winner, Huang the Condorcet neutral, and Jaw the Condorcet loser. Table 1 is the preference profile of the respondents who expressed strict preferences over the three candidates.

Table 1: Taipei City Respondents' Preference Orderings⁵

(30.9%)	(11.6%)	(16%)	(18.7%)	(12.6%)	(10.2%)
C	C	J	J	H	H

ranked first by a majority of the voters, which contradicts our assumption that no candidate receives more than fifty percent of the votes.

⁵ Respondents (5.5%) who indicated indifference between two of the candidates or among all three candidates are not included in this table.

H	J	H	C	C	J	
J	H		C	H	J	C

Chen was the leading candidate from the beginning of the race. To prevent either of his competitors from gaining momentum, he tried to maintain a balance of power between Jaw and Huang by directing attacks at whomever was moving ahead. The two trailing candidates were competing hard for the No. 2 position, hoping to capture strategic votes. Three days before the election, the convener of the New Party, Wang Chien-hsuan, unexpectedly held a press conference and announced that he had received information from reliable sources that President Lee Teng-hui, who was also the KMT chairman and Huang's mentor, had decided "to dump Huang to save Chen." The press conference was covered by the national TV networks and viewed by millions of voters. The primary effect of this statement was to signal to voters that Jaw's chance of winning had surpassed that of Huang, implying that Huang's supporters should consider voting for their second most-preferred candidate. The press conference, to a large extent, derailed Huang's campaign and a large number of Huang's supporters voted strategically for Chen or Jaw (Hsieh, Niou, and Paolino, 1997).

The outcome of the election was not affected by the strategic voting of Huang's supporters. Chen Shui-bian, the plurality and the Condorcet winner, won the election by garnering 43.7% of the vote. The runner-up, Jaw Shau-kong, receiving 30.2% of the vote,. Ta-chou Huang received 25.9% of the vote.

Remark 1b is supported by the case in India. According to Riker (1976), the Congress party is long-lived because it is the plurality and the Condorcet winner: "Congress, the largest party, is squarely in the middle. To the left are socialists and

several versions of communists, and to the right are religious and economic conservatives. Congress in the center has usually been able to keep the opposite ends from combining against it, except on a few occasions in the state parliaments (p.104).” An implicit assumption in Riker’s argument is that voters in India have single peaked preference. Therefore, strategic voting was not only inconsequential, no voters had the incentives to vote strategically because n_4 and n_6 were empty.

Cases 2a & 2b: The Plurality Winner is the Condorcet Neutral

In the case of 2a, since candidate b is the Condorcet winner, $n(3,4,6) > n(1,2,5)$, n_6 can vote strategically for b to render the Condorcet winner, b, the plurality winner. Bloc n_4 is not in a position to vote strategically, however, because c is the Condorcet loser, $n(1,2,3) > n(4,5,6)$. In the case of 2b, by the same token, only n_4 can vote strategically for c to render the Condorcet winner, c, the plurality winner.

Remark 2: In three-candidate elections, under simple plurality rule, if the plurality winner is the Condorcet neutral, then the supporters of the Condorcet loser who rank the plurality winner last will help the Condorcet winner prevail by voting strategically.

Remark 2 does not change if we assume single-peaked preferences. By Lemma 1 (footnote 3), in the case of 2a, candidate b is positioned in between a and c, which means that $n(2)=n(5)=0$. In the case of 2b, candidate c is in between a and b and $n(1)=n(3)=0$. Neither case excludes the possibility that voters have preference rankings $b > c > a$ and $c > b > a$.

Dummett (1984) offers the same argument as Remark 2 in his discussion of strategic voting under the alternative vote method: “Advocates of the procedure tend to write as though the only ground for believing an outcome to have no chance is the knowledge that it has few committed supporters, making it likely to be eliminated at stage 1. This is obviously untrue: one may know of an outcome that, although it has significantly many supporters, there is also strong opposition to it, dooming it to defeat at a later stage.” (p.228) To illustrate, in Example 3, the plurality winner, a, is the Condorcet neutral, the second highest vote getter, b, is the Condorcet loser, and the candidate who has the lowest first-place vote, c, is the Condorcet winner. If n4 votes strategically for c, c wins. Bloc n3 cannot counter n4’s move because $n(4,5,6) > n(1,2,3)$.

Example 3

n1	n2	n3	n4	n5	n6
a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a
10	30	5	30	15	10

Social Preference Ordering: $c > a > b$

The 1964 Chilean presidential election provides empirical support for Remark 2. In that election, the Right (the Liberals and Conservatives), believing that they were unlikely to find a viable candidate to compete against Salvador Allende, the candidate of a united Left, decided to endorse the candidate supported by the Center, Eduardo Frei.⁶ Frei

⁶ This belief was based on the fact that a candidate presented by the alliance of Radicals, Liberals, and the Conservatives lost a crucial by-election to the Left in months before the 1964 presidential contest (Valenzuela 1978).

proceeded to win the election by a comfortable margin of 56.1% of the vote to 38,9% for Allende. This example shows that Chilean politicians and voters acted strategically, by throwing their support to the Condorcet winner, Frei, to prevent the Condorcet loser, Allende, from winning the 1964 presidential election.⁷

Cases 3a & 3b: The Plurality Winner is the Condorcet Loser

If the plurality winner, a , is the Condorcet loser, then candidate a can be defeated if either n_4 or n_6 votes strategically for their second best candidate. However, it is not clear whether it is n_4 or n_6 who should vote strategically. To illustrate, in Example 4a, the plurality winner, a , is the Condorcet loser and b the Condorcet winner. If no one votes strategically, a wins. If only n_4 votes strategically for its second best candidate, c , c wins. If only n_6 votes strategically, b wins. But if both n_4 and n_6 vote for their second best candidates, then a wins. This example shows that n_4 and n_6 are actually playing the battle-of-the-sexes game. The outcome of the game is indeterminate.

Moreover, when the plurality winner is the Condorcet loser, the strategic voting outcome may depend not only on the voting decision by n_4 and n_6 ; moves by n_3 and n_5 might also affect the voting outcome. To illustrate, suppose $b > c > a$ ($n(3,4,6) > n(1,2,5)$ and $n(4,5,6) > n(1,2,3)$) and $n(3,6) > n(1,2)$. Since $n(3,6) > n(1,2)$, when n_4 and n_6 are playing the two-person game, n_4 's dominant strategy is to vote for c . Because regardless of whether n_4 supports b , n_3 and n_6 together can render b the winner if n_6 chooses to

⁷ In the 1958 Chilean presidential election, Jorge Alessandri, the candidate supported by the Right, won the race by receiving 31.2% of the vote while the Condorcet loser Salvador Allende, supported by the united Left, received 28.5% of the vote. Actually, Eduardo Frei, supported by the Center, receiving 20.5% of the vote, was more likely the Condorcet winner (Valenzuela, 1978). The Left probably did not support the Center because the race between Alessandri and Allende was so close and they mistakenly thought they could succeed on their own.

vote strategically for b. Correspondingly, n6's dominant strategy is to vote for c. c wins. By anticipating this outcome, n3 should pre-commit to vote for a. If $n(4,6) < n(1,2,3)$, n3's preemptive move would not affect the outcome (see Example 4b). After n3 precommits to vote for a, in the two-player game between n4 and n6, b has no chance of winning because voting for b becomes a dominated strategy for n6. So n4's dominant strategy is to vote for c. c wins because $n(4,5,6) > n(1,2,3)$. But if $n(4,6) > n(1,2,3)$ and if commitments are credible, n3's preemptive move to vote for a converts the two-player game between n4 and n6 into the battle-of-the-sexes game. The strategic voting outcome becomes indeterminate. In addition, n5's attitude toward risk might affect the outcome. If n5 is risk-averse or if n5 thinks that its least preferred candidate, b, is a lot worse than the other two candidates, then n5 might also pre-commit to vote for a. n5 can secure a if $n(4,6) < n(1,2,3,5)$ (see Example 4c), otherwise, n4 and n6 will still be playing the battle-of-the-sexes game (see Example 4d).

Examples 4a-4e

n1	n2	n3	n4	n5	n6	
a	a	b	b	c	c	
b	c	a	c	a	b	
c	b	c	a	b	a	
21	0	0	20	0	19	(4a)
11	10	8	12	3	16	(4b)
11	10	7	13	3	16	(4c)
11	10	7	13	1	18	(4d)

Social Preference Ordering: $b > c > a$

The argument for strategic voting when $n(4,5) > n(1,2)$ is the same as the one given when $n(3,6) > n(1,2)$. In this case, n_6 becomes the voting bloc that has a dominant strategy in the two-player game and n_5 the voting bloc that has incentives to preempt n_6 's move.

When $n(3,6) < n(1,2)$ and $n(4,5) < n(1,2)$, neither n_4 nor n_6 has a dominant strategy. In this case, n_4 and n_6 are playing the battle-of-the-sexes game. Does either n_3 or n_5 have incentives to pre-commit to vote for a ? If $n(1,2,3) < n(4,6)$, then n_3 does not carry enough weight to influence the outcome by voting for a . If $n(1,2,3) > n(4,6)$ and if n_3 pre-commits to vote for a , b cannot be the outcome of the two-player game between n_4 and n_6 even if n_6 votes for it. Then n_4 's best response is to vote for c . c wins because $n(4,5,6) > n(1,2,3)$. Since c is n_3 's least preferred outcome, it is not in n_3 's interest to vote for a in the first place. By the same token, n_5 should not pre-commit to vote for a . Candidate a can win only if $n(1,2,3,5) > n(4,6)$ and n_3 and n_5 simultaneously pre-commit to vote for a (see Example 4e). Otherwise, the outcome is indeterminate.

It is not possible to have $n(4,5) > n(1,2)$ and $n(3,6) > n(1,2)$ because it is assumed that a is the plurality winner, meaning $n(1,2) > n(3,4)$ and $n(1,2) > n(5,6)$. So we do not need to consider this possibility.

We can summarize the conditions for strategic voting discussed above in the following remark.

Remark 3a: In three-candidate elections, under simple plurality rule, if the plurality winner is the Condorcet loser and if commitments are credible, the strategic voting outcome is

- (1) b if $n(4,5) > n(1,2)$ and $n(4,6) < n(1,2,5)$,
- (2) c if $n(3,6) > n(1,2)$ and $n(4,6) < n(1,2,3)$,
- (3) a if $n(4,5) > n(1,2)$, $n(1,2,3,5) > n(4,6) > n(1,2,5)$, and n_3 prefers a over the expected outcome of the battle-of-the-sexes game,
- (4) a if $n(3,6) > n(1,2)$, $n(1,2,3,5) > n(4,6) > n(1,2,3)$ and n_5 prefers a over the expected outcome of the battle-of-the-sexes game,
- (5) a if $n(1,2,3,5) > n(4,6)$, $n(4,5) < n(1,2)$, $n(3,6) < n(1,2)$, and n_3 and n_5 simultaneously pre-commit to vote for a.
- (6) otherwise, the outcome is indeterminate.

Suppose candidate a is the Condorcet loser and commitments are not credible. If $n(3,6) > n(1,2)$, n_3 cannot credibly commit to vote for a. n_4 's and n_6 's dominant strategy is to vote for c. c wins. If $n(4,5) > n(1,2)$, n_5 cannot commit credibly to vote for a. n_4 's and n_6 's dominant strategy is to vote for b. b wins. If $n(3,6) < n(1,2)$ and $n(4,5) < n(1,2)$, then n_4 and n_6 will be playing the battle-of-the-sexes game. The outcome of the game is indeterminate. Therefore, the outcome of the strategic voting game depends on the signs of the two inequalities.

The two inequalities, $n(3,6) > n(1,2)$ and $n(4,5) > n(1,2)$, imply how unified the supporters of each of the two trailing candidates are. The inequality $n(3,6) > n(1,2)$

implies that the supporters of candidate b are spread more evenly across n3 and n4 but the supporters of candidate c are more concentrated on n6.⁸ The inequality $n(4,5) > n(1,2)$ implies that the supporters of b are more concentrated on n4 but the supporters of c are more evenly divided between n5 and n6. The inequalities $n(3,6) < n(1,2)$ and $n(4,5) < n(1,2)$ imply that the supporters of the two candidates are dispersed across two blocs or are both concentrated in a single bloc. The lesson we can draw from the strategic voting outcomes is that if the supporters of one trailing candidate are more concentrated than the supporters of the other trailing candidate, then the former candidate has a better chance of winning than the latter. When the supporters of the two trailing candidates are both concentrated or both dispersed, then the outcome of the game becomes indeterminate.

Remark 3b: In three-candidate elections, under simple plurality rule, if the plurality winner is the Condorcet loser and if commitments are not credible, the strategic voting outcome is b if $n(4,5) > n(1,2)$, c if $n(3,6) > n(1,2)$, and indeterminate if neither inequality is satisfied.

If we assume single-peaked preferences, and if b is the Condorcet winner, by Lemma 1, $n(2)=n(5)=0$, which implies that $n(4,5) < n(1,2)$. If c is the Condorcet winner, $n(1)=n(3)=0$, which implies that $n(3,6) < n(1,2)$. So, when the Condorcet loser is the plurality winner, regardless whether commitments are credible, the Condorcet winner can

⁸ Suppose the contrary is true. The supporters of b are relatively unified, and n4 is a small number. Since a is the Condorcet loser, $n(4,5,6) > n(1,2,3)$. If n4 has relatively few voters, $n(5,6)$ cannot be greater than $n(1,2,3)$ since a is the plurality winner, $n(1,2) > n(5,6)$. Next suppose the supporters of c are not unified. That is n5 is not a small number. Since a is the Condorcet loser and the plurality winner, $n(3,4,6) > n(1,2,5)$ and $n(1,2) > n(3,4)$. So n6 must be significantly greater than n5.

never be the deterministic strategic voting outcome if we assume single-peaked preferences. This is because, if we assume single-peaked preference, the supporters of the Condorcet neutral are concentrated in a single voting bloc, which makes it impossible for the Condorcet winner to be the deterministic strategic voting outcome. This is an interesting result because, in this case, single-peaked preferences do not contribute to the selection of the Condorcet winner.

Remark 3c: In three-candidate elections, under simple plurality rule, if the plurality winner is the Condorcet loser and if we assume single-peaked preferences, the Condorcet winner will never be the deterministic strategic voting outcome.

The 1970 Chilean presidential election is often cited as an example in which the plurality winner, Salvador Allende, was the Condorcet loser. Allende, receiving 36.2% of the vote, was the candidate for the united Left (Socialists and Communists); Radomiro Tomic, receiving 27.8% of the vote, was the candidate for the Center (Christian Democrats); and Jorge Alessandri, receiving 34.9% of the vote, was supported by the Right (Nationals). Since “the Christian Democrats as a group were probably closer in ideological distance to the National party as a whole,”⁹ Allende simply would not have received a majority of the vote in a two-way race. But the Center and the Right were engaged in the battle of the sexes game and they failed to reach a compromise.

The 1987 and 1997 presidential elections in South Korea present other examples of Condorcet losers winning the elections. Although no reliable information is available to construct the preference ranking of voters over the three candidates in the 1987 election,

T. W. Roh, Y. S. Kim, and D. J. Kim, it is commonly believed that Roh was the Condorcet loser. Between the two Kims, Y. S. was more popular than D. J. If either Y. S. Kim or D. J. Kim had dropped out of the race, Roh could not have won the presidential election. In the 1997 presidential elections, among the three candidates, D. J. Kim, H. C. Lee, and I. J. Lee, again, it is commonly believed that the plurality winner D. J. Kim was the Condorcet loser and H. C. Lee was the Condorcet winner.

Cases 4a & 4b: The Condorcet Winner Does Not Exist

When the Condorcet winner does not exist, by definition, the plurality winner can be defeated by one of the trailing candidates. For example, if $a > b > c > a$, then $n(4,5,6) > n(1,2,3)$ and $n(3,4,6) < n(1,2,5)$. So in this case only n_4 is in the position to vote strategically for c to offset a . If no voting bloc can make credible commitments, then c will be the outcome of the game. If we assume that blocs can make credible commitments, n_1 , by anticipating that c would be the outcome, has the incentives to commit to support b . But then, n_5 has the incentives to commit to support a . Then n_4 should vote for c , and so on. Therefore, by allowing voting blocs to make credible commitments, we transform the game into the three-person divide the dollar game in which the core is empty and no outcome is stable (Ordeshook 1986).

Remark 4: In three-candidate elections, when the Condorcet winner does not exist, if blocs cannot make credible commitments, the candidate who is preferred to the plurality winner in the pairwise comparison will be the strategic voting winner. Those voters whose least preferred candidate is the plurality winner and whose most preferred candidate loses

⁹ Arturo Valenzuela made this argument in *The Breakdown of Democratic Regimes: Chile, 1978*, by

to the plurality winner in the pairwise comparison will vote strategically. If we assume credible commitments, then the strategic voting outcome is indeterminate.

I summarize Remarks 1a-4b in Tables 4 and 5.

Table 4

Strategic Voting under Simple Plurality Rule
with Credible Commitment

		Strategic Voting by Which Groups	Winner
1a	$a > b > c$	none	a
1b	$a > c > b$	none	a
2a	$b > a > c$	n6	b
2b	$c > a > b$	n4	c
3a	$b > c > a$	n3, n4, n5, or n6	a, b, c, or indeterminate
3b	$c > b > a$	n3, n4, n5, or n6	a, b, c, or indeterminate
4a	$a > b > c > a$	n1, n4, or n5	indeterminate
4b	$a > c > b > a$	n2, n3, or n6	indeterminate

Table 5

Strategic Voting under Simple Plurality Rule
without credible commitment

		Strategic Voting by Which Groups	Winner
1a	$a > b > c$	none	a
1b	$a > c > b$	none	a
2a	$b > a > c$	n6	b
2b	$c > a > b$	n4	c
3a	$b > c > a$	n4 or n6	b, c, or indeterminate
3b	$c > b > a$	n4 or n6	b, c, or indeterminate
4a	$a > b > c > a$	n4	c

relying on some survey data.

4b a > c > b > a n6 b

4. Strategic Voting Equilibrium under the Runoff Rule

Thus far, I have shown that the condition under which strategic voting can alter the outcome under simple plurality rule is when the plurality winner is not the Condorcet winner, that is, in cases 2a, 2b, 3a, 3b, 4a and 4b. Under the runoff system, if no one votes strategically in the first round, the plurality winner does not automatically win the election.¹⁰ If we assume that the supporters of the last place finisher, n5 and n6, will not abstain from voting in the second round and that they will vote for their second most preferred candidates, then between the top two finishers in the first round, the one supported by a majority of the voters will prevail in the second round. In this circumstance, voting blocs n1 and n3 have no incentives to vote strategically for their second most preferred candidates because by doing that they simply add to the margin by which the other candidate won. Blocs n2 or n4 will choose to vote for c if their most preferred candidate will lose in the second round. Blocs n5 and n6 need not vote strategically for their second most preferred candidates in the first round because they will vote for them in the second round after c is eliminated in the first round. So under the runoff rule, the only blocs of voters that might have incentives to vote strategically in the first round are n2 and n4. In the following, I analyze the strategic voting equilibrium under the runoff rule for each of the eight possible preference profiles.

¹⁰ In this paper, I assume that only the top 2 finishers in the first round advance to the second round.

Cases 1a, 1b, 2a, and 3a: The Condorcet Winner is the First or the Second Place Finisher in the First Round

If the plurality winner is also the Condorcet winner, then n_2 clearly has no incentives to vote strategically and n_4 cannot change the election outcome by voting strategically for candidate c in the first round because c will lose to a in the second round. If the Condorcet winner is the second place finisher in the first round, n_4 has no incentives to vote strategically for c because candidate b will prevail in the second round. Bloc n_2 cannot change the election outcome by voting strategically for candidate c in the first round because c will lose to b in the second round. That is, as long as the Condorcet winner is not eliminated in the first round, the Condorcet winner will prevail in the second round.

Remark 5: In three-candidate elections, under the runoff rule, if the Condorcet winner is the first or the second place finisher in the first round, then strategic voting is inconsequential and the Condorcet winner will win.

Cases 2b and 3b: The Condorcet winner is the Last Place Finisher in the First Round

If the Condorcet winner is the last place finisher in the first round, then n_2 will vote for c if a will lose to b in the second round or n_4 will vote for c if b is to lose to a in the second round. In either case, among the supporters of the Condorcet loser, those who rank the Condorcet winner as the second most preferred candidate will help the Condorcet winner prevail by voting strategically.

Remark 6: In three-candidate elections, under the runoff rule, if the Condorcet winner is the last place finisher in the first round, then among the supporters of the Condorcet loser, those who rank the Condorcet winner as the second most preferred candidate will help the Condorcet winner prevail by voting strategically.

Cases 4a and 4b: The Condorcet Winner Does Not Exist.

In three-candidate elections, under the runoff rule, if the Condorcet winner does not exist and if no blocs vote strategically, among the top two finishers in the first round, the one preferred by a majority in the pairwise comparison will prevail in the second round. If blocs cannot make credible commitments, then n_2 will vote for c if the social preference order is $b > a > c > b$ and n_4 will vote for c if the social preference order is $a > b > c > a$. If we assume credible commitments, then the strategic voting outcome becomes indeterminate. To illustrate, for example, if the social preference ordering is $a > b > c > a$, n_1 can preempt n_4 's move to vote for c by committing to vote for b . n_5 , however, can counter n_1 's move by committing to vote for a , and so on.

Remark 7: In three-candidate elections, under the runoff rule, if the Condorcet winner does not exist and if commitments are not credible, then n_4 will vote for c if the social preference order is $a > b > c > a$ and n_2 will vote for c if the social preference order is $b > a > c > b$. If commitments are credible, the strategic voting outcome becomes indeterminate.

Duverger claims that the runoff rule produces no incentives to vote strategically in the first round: “the variety of parties having much in common does not adversely affect the total number of seats they gain since in this system they can always regroup for the second ballot” (Duverger 1954: 240). Remarks 6 and 7, however, show that under the runoff rule, strategic voting matters in the first round of the election if the Condorcet winner is the last place finisher (cases 2b and 3b) or if the Condorcet winner does not exist (cases 4a and 4b).

In Table 6, I summarize the findings on strategic voting under the runoff rule.

Table 6

Strategic Voting under Runoff Rule

		Strategic Voting by Which Groups	Winner
1a	$a > b > c$	none	a
1b	$a > c > b$	none	a
2a	$b > a > c$	none	b
2b	$c > a > b$	n4	c
3a	$b > c > a$	none	b
3b	$c > b > a$	n2	c
4a	$a > b > c > a$	n4	c
4b	$a > c > b > a$	n6	b

An interesting finding revealed in Table 6 is that the Condorcet winner always wins if we assume strategic voting and complete information.

Remark 8: In three-candidate elections, under the runoff rule, if we assume that voters vote strategically and that voters have complete information, then the Condorcet winner always wins when one exists.

An interesting comparison between simple plurality rule and the runoff rule is the likelihood of the occurrence of strategic voting under the two systems. Under simple plurality rule, strategic voting occurs in six of the eight possible cases while under the runoff rule, strategic voting occurs in four out of the eight possible cases. If we assume that the eight types of preference profiles are equally likely to occur, then we can conclude that strategic voting is more likely to occur in elections under simple plurality rule than in elections under the runoff rule.

Remark 9: If every preference profile occurs with equal probability, then strategic voting is more likely to occur in elections under simple plurality rule than under runoff rule.

This finding is consistent with Cox (1997). However, the explanations are different. Remark 9 is true because strategic voting under the runoff rule matters only in four out of the eight preference profiles, but under simple plurality system, strategic voting matters in six out of the eight preference profiles. In other words, under the runoff procedure, strategic voting is rarer because there is less of a need to vote strategically. In Cox's model, however, strategic voting is less likely to occur under the runoff rules because "it is more difficult for elite actors to discern when it is in their interests to foment strategic voting, and--conditional on their deciding that it is worthwhile--it is more difficult to implement the appropriate strategy." (p.137)

It has been established previously by many scholars that strategic voting helps reduce the number of political parties (Duverger 1954, Cox 1987, 1997, Palfrey 1989, Myerson and Weber 1993). By showing that strategic voting is more likely to occur in elections under simple plurality rule than under the runoff rule, Remark 9 supports the

Duverger's argument that the number of political parties will be smaller under simple plurality rule than under the runoff rule.

5. Conclusion

This paper establishes the conditions under which strategic voting can change the election outcome and identifies the voters who should vote strategically under simple plurality rule and the runoff rule. Three interesting findings can be drawn from the results. First, under simple plurality rule, strategic voting can change the election outcome if the plurality winner is not the Condorcet winner or if the Condorcet winner does not exist; under the runoff rule, strategic voting can change the election outcome if the Condorcet winner is the plurality loser or if the Condorcet winner does not exist. Second, under simple plurality rule, the Condorcet winner is not always the strategic voting outcome even if we assume that voters have complete information about the preference profile. Even worse, the Condorcet loser could win. Under the runoff rule, however, strategic voting ensures that the Condorcet winner will win if one exists. Third, strategic voting is less likely to occur under the runoff rule than under simple plurality rule.

REFERENCES

- Abramson, Paul R., John H. Aldrich, Philip Paolino, David W. Rohde. 1995. "Third-Party and Independent Candidates in American Politics: Wallace, Anderson, and Perot." *Political Science Quarterly* 110:349-367.
- Cox, Gary W. 1987. "Duverger's Law and Strategic Voting." Unpublished manuscript, University of California, San Diego.
- Cox, Gary W. 1994. "Strategic Voting Equilibria under the Single Non-Transferable Vote." *American Political Science Review* 88:608-621.
- Cox, Gary W. 1997. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. New York: Cambridge University Press.
- Downs, Anthony. 1957. *An Economic Theory of Democracy*. New York: Harper and Row.
- Duverger, Maurice. 1954. *Political Parties*. New York: Wiley.
- Dummett, Michael 1984. *Voting Procedures*. Oxford University Press.
- Farquharson, Robin. 1969. *Theory of Voting*. New Haven: Yale University Press.
- Hsieh, John Fuh-sheng, Emerson M. S. Niou, and Philip Paolino. 1997. "Strategic Voting in the 1994 Taipei City Mayoral Election," *Electoral Studies*.
- Myerson, Roger and Robert Weber. 1993. "A Theory of Voting Equilibria." *American Political Science Review* 87:102-14.
- Niemi, Richard G. and Arthur Q. Frank. 1981. "Sophisticated Voting Under the Plurality Procedure." In *Political Equilibrium*, ed. Peter C. Ordeshook and Kenneth A. Shepsle, Kluwer-Nijhoff Publishing.
- Ordeshook, Peter C. 1986. *Game Theory and Political Theory*, Cambridge University Press.
- Palfrey, Thomas. 1989. "A Mathematical Proof of Duverger's Law." In *Models of Strategic Choice in Politics*, ed. Peter C. Ordeshook. Ann Arbor: University of Michigan Press.
- Riker, William H. 1976. "The Number of Political Parties: A Reexamination of Duverger's Law," *Comparative Politics*, 9:93-106.
- Valenzuela, Arturo. 1978. *The Breakdown of Democratic Regimes: Chile*. Baltimore: The Johns Hopkins University Press.